

Baryonic resonances mass spectrum from a modified perturbative QCD

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Abstract

A recently proposed modified perturbation expansion for QCD is employed to evaluate the quark self-energies. Results of the order of $1/3$ of the nucleon mass are obtained for the effective masses of the up and down quarks in a first approximation. Also, the predicted flavor dependence of the calculated quarks masses turns out to be the appropriate to well reproduce the spectrum of the ground states within the various groups of hadronic resonances through the simple addition of the evaluated constituent quark masses. The results suggests to conjecture that the modified expansion, after also introducing quark condensates in a same token as the gluonic ones, would be able to furnish a natural explanation of the mass spectrum of the three generations of fundamental fermions.

Strong interactions at low energy are the subject of an intense research activity directed to complete the present understanding of QCD [1, 2]. Two of the many important questions to be clarified are the nature of the mass spectrum for the numerous hadron and meson resonances and a better understanding of the differences between the constituent and Lagrangian quark masses [3, 4, 5]. On a more general level, there is also a little understanding of the physics determining the mass spectrum of the three generations of fundamental fermions [3].

In previous papers we have proposed a modification of the standard perturbative expansion for QCD which is based in the idea that the low energy properties are determined by the presence of a condensate of gluon pairs [6, 7, 8]. In paper [6] the general structure of the modified Feynman rules were proposed and showed that it produces a non-vanishing value of $\langle G^2 \rangle$ even in the simplest approximation and that the condensate seems to be generated spontaneously from the perturbative vacuum of zero particles in a similar form as the earlier Savvidy chromomagnetic vacuum field. Further, in paper [7, 8] the state of the non-interacting theory which shows the proposed kind of modified Feynman rules, was determined. It occurred that the state is constructed as a coherent superposition of zero momentum gluon pairs. For its determination the condition of being a physical state (a zero mode of the BRST charge) of the non-interacting theory was imposed. After that, the operational quantization was considered in a scheme of the Gupta-Bleuler type in which all the gluons are considered on the same footing [15]. This corresponds with the selection $\alpha = 1$ value of the gauge parameter in the standard functional approach. In justifying to proceed in such a way, our central assumption is that under the adiabatic connection of the color interaction, the evolution will not bring the state out of the physical subspace at any stage of the connection. Thus, the final state will be also a physical state of the interacting theory in which the associated Feynman expansion should have a physical meaning. It is clear that this approach leaves out the question of the construction of a gauge parameter independent formulation of the theory. However, as a minimal logical ground for the physical relevance of the predictions is given, we decided to postpone this more technical question to further analysis to be done.

Therefore, in the present work the proposed expansion is applied to evaluate the mass spectrum of the quarks as modified by the inclusion of the gluon condensation effects. The main result which follows is that a particular branch of the spectrum produces constituent mass values of the order

1/3 of the nucleon mass for the up and down quarks. Further, the result for flavor dependence of the effective quark masses is able to predict the rest masses of many of the ground state resonances in the various groups of them as classified in [10] after a simple addition of the values associated to the corresponding constituent quarks. It should be mentioned that the constituent quark predictions of the order of 1/3 of the nucleon mass as determined by gluon condensation effects were also obtained in [16, 17] by employing a different approach. Furthermore, the spectrum of mesons, which are currently accepted to show constituent quarks structure are also predicted. In our view, these results strongly support the important role of gluon condensation in determining the structure of many hadrons and mesons and also the possibility of describing them with a modified perturbative expansion. If such is the case, various non-perturbative characteristics of the QCD could be treated in a similar way as in the theory of Bose Condensation [11]. The non-perturbative properties of the theory, then, would be described by the condensation parameter controlling the modified expansion. The possibility for such a picture was already suggested by the earlier chromomagnetic field models [12, 13, 14]. It is natural to expect that the Lorentz invariance of the ground state and the vector character of the gluonic field makes it natural that a Bose condensate effect for gluonic field should present peculiar characteristics as in the case of superconductivity [11]. However, this situation is not excluding the possible applicability of a modified perturbative expansion as in standard Bose condensed systems [11]. The results for the mass spectra arising from the calculations shown here support the existence of such a picture.

The alternative initial state determining the modified perturbation expansion through the Wick theorem was found in [7, 8] to have the form

$$|\phi\rangle = \exp \sum_a \left(\frac{1}{2} A_{0,1}^{a+} A_{0,1}^{a+} + \frac{1}{2} A_{0,2}^{a+} A_{0,2}^{a+} + B_0^{a+} A_0^{L,a+} + i \bar{c}_0^{a+} c_0^{a+} \right) |0\rangle \quad (1)$$

in terms of the transverse and longitudinal gluon $A_{0,1}^{a+}, A_{0,2}^{a+}, A_0^{L,a+}$, ghosts c_0^{a+}, \bar{c}_0^{a+} , and Nakanishi-Lautrup B_0^{a+} creation operators of the interaction free theory [15]. The state is colorless as indicated by the contracted color index a . For its construction the BRST physical state conditions $Q_B |\Phi\rangle = 0$ and $Q_C |\Phi\rangle = 0$ were required, in which Q_B and Q_C are the charges associated to the BRST symmetry and ghost number conservation.

In Refs. [7, 8] the state (1) was sought in order to implement that the

net effect of the application of the Wick Theorem to the expansion of the evolution operator be to produce the form of the gluon propagator introduced in [6]. The expression of the modified propagator in the momentum representation is

$$G_{0\mu\nu}^{ab}(k) = \delta^{ab} g_{\mu\nu} \left[\frac{1}{k^2} - iC\delta(k) \right], \quad (2)$$

in which C is the parameter associated with the gluon condensate, and was determined to be real and nonnegative [7, 8]. The ghost and fermion propagators do not showed a modification. It will be the case that under assuming the existence of fermion condensates such a modification should appear. Precisely this possibility leads to the conjecture to be posed in the concluding remarks.

In order to fix a physically supported value for the parameter C , the following recourse was employed [6]. The gluon condensate parameter $\langle G^2 \rangle$ was calculated up to the order g^2 and the resulting function of the parameter C used to determine this constant by selecting its value to give the presently accepted estimate of $\langle g^2 G^2 \rangle$ in the physical vacuum. The details of this evaluation are presented in an extended version of this article [18]. For the calculation of $\langle g^2 G^2 \rangle$ the following expression was employed

$$\langle 0 | S_g | 0 \rangle = \left[\frac{1}{N} \int D\phi S_g[\phi] \exp(S_T(\phi)) \right],$$

where $S_g[\phi]$ represents the usual gauge invariant gluon part of the action and S_T is the total Fadeev-Popov action for $\alpha = 1$.

The final result for $\langle g^2 G^2 \rangle$ in terms of C is

$$\langle g^2 G^2 \rangle = \frac{288g^4 C^2}{(2\pi)^8}.$$

Further, by making use of the estimated value for $\langle g^2 G^2 \rangle \cong 0.5 (GeV/c^2)^4$ [1], is obtained the result $g^2 C = 64.9394 (GeV/c^2)^2$.

After the phenomenological determination of $g^2 C$, let us consider in what follows the one loop correction to the quark self-energy. The notation employed is the one in Ref. [9]. The inverse quark propagator has the form

$$G_{2ij}^{-1}(p) = i\delta_{ij} (m_Q - p^\mu \gamma_\mu - \Sigma(p)), \quad (3)$$

in which the self-energy part $\Sigma(p)$ is determined up to the order g^2 and m_Q is the Lagrangian mass of the specific kind of flavor being considered.

The only change appearing in (3) with respect to the similar calculation in the standard perturbation expansion is related with the gluon propagator $G_{0\mu\nu}^{ab}(k)$ to be used which will include the mentioned condensate term as defined in (2). Here we will be concerned only with the evaluation of the contribution to the self-energy of the condensate term. The aim is to investigate within the most simple approximation the predictions for the mass corrections determined by it. Therefore, after integrating in the expression for $\Sigma(p)$ it follows for the inverse Green function (3)

$$G_{2ij}^{-1}(p) = i\delta_{ij} \left(m_Q \left(1 + 2 \frac{M^2}{(m_Q^2 - p^2)} \right) - p^\mu \gamma_\mu \left(1 + \frac{M^2}{(m_Q^2 - p^2)} \right) \right), \quad (4)$$

where $M^2 = 0.111111 \text{ (GeV/c}^2\text{)}^2$.

The vanishing of the determinant of the matrix (4) determines the modified mass shell in the considered approximation including the effects of the condensate on the spectrum. The mass values will depend on the quark flavor through their Lagrangian mass parameters m_Q which have been determined by independent means [3].

After solving the cubic equation produced by equating to zero the determinant of (4), the following quark mass shells relations are obtained

$$p^2 - m_{q,l}^2 = 0; \quad \text{for } l = 1, 2, 3$$

in which $m_{q,l}^2, l = 1, 2, 3$ designates the three different analytic solutions for the squared masses written below:

$$\begin{aligned} m_{q,1}^2 &= A^{\frac{1}{3}} - B + m_Q^2 + \frac{2}{3}M^2, \\ m_{q,2}^2 &= -\frac{1}{2}A^{\frac{1}{3}} + \frac{1}{2}B + m_Q^2 + \frac{2}{3}M^2 + \frac{1}{2}i\sqrt{3} \left(A^{\frac{1}{3}} + B \right), \\ m_{q,3}^2 &= -\frac{1}{2}A^{\frac{1}{3}} + \frac{1}{2}B + m_Q^2 + \frac{2}{3}M^2 - \frac{1}{2}i\sqrt{3} \left(A^{\frac{1}{3}} + B \right), \end{aligned}$$

where the quantities A, B are determined by the expressions

$$\begin{aligned} A &= \frac{5}{6}m_Q^2M^4 - \frac{1}{27}M^6 + \frac{1}{18} \left(96m_Q^6M^6 + 177m_Q^4M^8 - 12m_Q^2M^{10} \right)^{\frac{1}{2}}, \\ B &= \frac{\frac{2}{3}m_Q^2M^2 - \frac{1}{9}M^4}{A^{\frac{1}{3}}}. \end{aligned}$$

The various dispersion relations arising from these solutions have some degree of redundancy. However, only six independent solutions emerge at the end for all of the $m_{q,l}^2$, with $l = 1, 2, 3$. The most of these six branches lead to imaginary values of the masses in some regions of the m_Q values. The physical nature of these fermionic wave modes deserve a closer examination which will be considered elsewhere. Here we limit ourselves to examine one of the solutions for the squared mass parameter which shows positive values m_q for the fermion squared masses at any momenta and also a growing dependence of the mass on the Lagrangian mass parameter m_Q . The plot of the graph m_q as function of m_Q for this solution is shown in Fig.1 in the region $m_Q < 2 \text{ GeV}/c^2$.

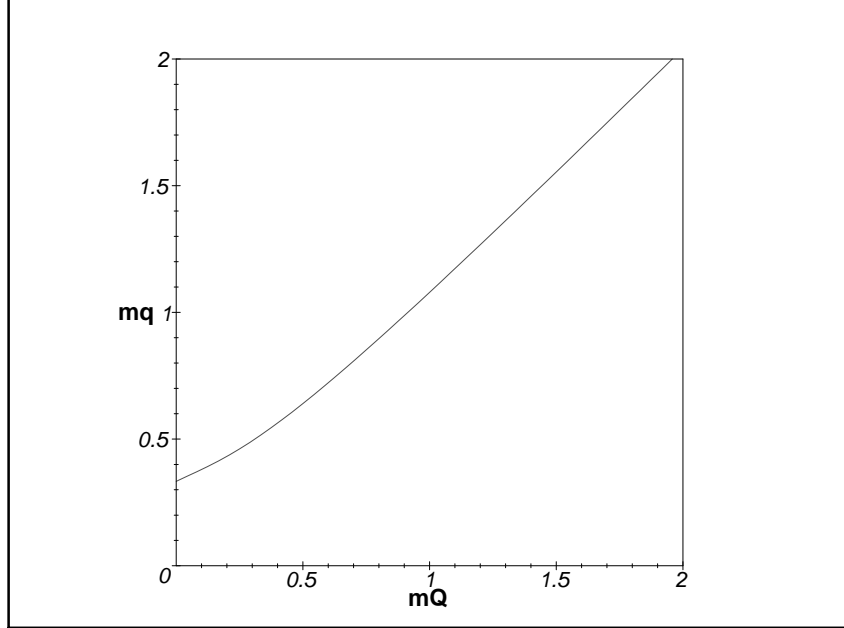


Figure 1: Real solution for the quark mass as a function of the Lagrangian mass (masses in units of GeV/c^2).

As it can be observed from Fig.1, the interaction with the vacuum condensate dress an initially assumed massless quark ($m_Q = 0$) by giving it a finite mass value of the order of one third of the nucleon mass. That is,

the u and d quarks acquire in this propagation mode a high contribution to their masses due to the interaction with the condensate. In Table 1 the mass values of the considered propagation mode for each of the six quark flavors as characterized by their reported Lagrangian mass values m_Q in Report [10] are shown.

Table 1: Quark mass values in presence of the condensate in units of MeV/c^2 .

<i>Quarks</i>	m_{Low}^{Exp}	m_{Up}^{Exp}	m_{Med}^{Theo}
(u)	1.5	5	334.944
(d)	3	9	336.287
(s)	60	170	388.191
(c)	1100	1400	1315.241
(b)	4100	4400	4269.572
(t)	168600	179000	173800.48

where m_{Low}^{Exp} is the reported lower bound value for the Lagrangian mass, m_{Up}^{Exp} is the reported upper value for the Lagrangian mass and m_{Med}^{Theo} is the calculated mean value of the constituent mass, for the lower and upper bound of the Lagrangian masses.

Table 1 shows that the masses of the u and d quarks are near one third of the nucleon mass. It can be noticed that the c, b and t quarks show masses for which the effect of gluon condensation is not neatly evidenced. The light quarks u, d and s are the ones which get a clear influence on their mass due to gluon condensation as described here.

These results support that the expansion introduced in Ref. [6, 7, 8] could have the chance of furnishing important non-perturbative information on the QCD low energy physics. This conclusion is further strengthened by evaluating the masses of the ground states within the various groups of baryonic resonances as classified in [10]. The calculation is done by simply adding the masses corresponding to each of the types of quarks constituting the specific hadron being examined. Table 2 shows the results of that evaluation for the lower energy states within the groups of baryon resonances in conjunction with their experimentally determined masses as given in [10].

Table 2: Experimental and Theoretical Baryonic Resonance Masses in units of MeV/c^2 .

Baryon	Exp.Val.	Th.Mean.Val.	Rel.Err.
p(uud)	938.27231	1006.175	7.24
n(udd)	939.56563	1007.519	7.23
Λ (uds)	1115.683	1059.422	5.04
Σ^+ (uus)	1189.37	1058.078	11.04
Σ^0 (uds)	1192.642	1059.422	11.17
Σ^- (dds)	1197.449	1060.766	11.41
Ξ^0 (uss)	1314.9	1111.325	15.48
Ξ^- (dss)	1321.32	1112.669	15.79
Ω^- (sss)	1642.45	1164.572	29.10
Λ_c^+ (udc)	2284.9	1986.472	13.07
Ξ_c^+ (usc)	2465.6	2038.375	17.33
Ξ_c^0 (dsc)	2470.3	2039.719	17.43
Ω_c^0 (ssc)	2704	2091.622	22.65
Λ_b^0 (udb)	5624	4940.803	12.15

As it can be noticed, the values in Table 2 reasonably well match the mass spectrum of the ground states in each of the families of baryons as classified in [10]. Let us comment about some possible sources of theoretical errors associated to the evaluation of the quark masses and the baryon spectrum. We estimate that one of them is the non inclusion of the standard one loop self energy contributions for the quarks. It should be stressed that its proper consideration within the momentum scale: $p < 1$ GeV, needs for a knowledge of the running coupling in this region. However, the influence of the modified propagator on the coupling reflects itself only through a two loop evaluation of this quantity. This is because, the one loop divergences are unchanged by the use of the new propagator. In connection with the relative influence of the appreciable lack of precision in the current masses it could be observed that: a) For the mainly massless u and d quarks, the results obtained can not be greatly influenced by the big errors in the estimates of current masses, b) For high values of the current masses for c, b, and t quarks the influence of the condensate can be expected to be weak and the prediction should coincide

with the current masses and c) On the other hand for the strange s quark, the effect of the condensate is of the same order as the current mass value. Thus, the error in the estimate of this magnitude could become relevant for the evaluation of the effective mass. This argument, might be working for the results for the baryons in Table II including u and d quarks plus one, two or three s quarks. An increase in the effective s quark mass of near $180 \text{ MeV}/c^2$ seems to very much improve the predictions for such resonances. Another relevant source of errors is the high margin of error of the present estimates for the current masses. Clearly, the interaction effects could have their influence in the results. Their consideration, however, requires a further study of the predictions for the bound state spectrum of quarks.

Next, the calculated masses for a group of vector mesons is depicted in Table 3 in conjunction with their reported experimental values [10]. The evaluation is performed again by simply adding the mass of the corresponding quarks entering in the known composition of each meson.

Table 3: Experimental and Theoretical Masses for a group of Vector Mesons in units of MeV/c^2 .

Meson	Exp.Val.	Th.Mean.Val.	Rel.Err.
$\rho \left(\frac{u\bar{u}-d\bar{d}}{\sqrt{2}} \right)$	770.0	671.231	12.83
$\omega \left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \right)$	781.94	671.231	14.16
$\phi (s\bar{s})$	1019.413	776.381	23.84
$J/\psi (1S) (c\bar{c})$	3096.88	2630.482	15.06
$Y (1S) (b\bar{b})$	9460.37	8539.144	9.74

As it can be observed from the results for the vector mesons and baryonic resonance masses, in spite of the simple approximation which has been considered, the general structure of the mass spectrum is well reproduced under the only assumption of the accepted value of the gluon condensation parameter $\langle g^2 G^2 \rangle$, [1].

It should underlined that among the obtained dispersion relations, there is a solution predicting a vanishing value of the mass when $m_Q \rightarrow 0$. It might be the case that these modes could be connected with the family of low mass mesons (e.g. $\Pi^{\pm,0}$ mesons). Specifically, the possibility exists that the bound

states of quark excitations in these low mass states of quarks could describe such low lying mesons. An analysis of this question however, will be deferred to a future extension of the work. As a last point, we would like to conjecture on a possibility suggested by the analysis given here. It is related with the question about whether the mass spectrum of the whole three generations of fundamental fermions could be predicted by a slight generalization of the modified perturbation theory under consideration. In this sense, the presented results led us to the idea that after the introduction of quark condensates along the same lines as it was done for gluon ones in [6, 7, 8], the obtained perturbation expansion can have the chance to predict both, the Lagrangian mass and the constituent quark mass spectra of the three families of fundamental fermions. The fermion condensates as described in the proposed perturbative way, would be encharged to produce the Lagrangian quark masses as usual, through the chiral symmetry breaking. The gluonic condensates, in one hand, and as illustrated here, could be responsible of generating states of large constituent mass for the low mass quarks states. In another hand, it seems feasible that the higher order radiative corrections (including color interactions with the condensate as well as chiral symmetry corrections) could also determine the mass spectra for leptons and neutrinos. The smaller scale for the masses of these particles could be produced by the lack of lower order color interaction terms in their self-energy. Therefore, the possibility that the Lagrangian mass spectrum of the three generations of the fundamental fermions could be predicted by a modified perturbation expansion of the sort being proposed is suggested. Work devoted to investigate the above conjecture will be considered elsewhere.

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References

- [1] E. U. Shuryak, *The QCD Vacuum, Hadrons and the Superdense Matter*, World Scientific, Singapore, 1988.

- [2] S. Narison, *QCD Spectral Summ Rules*, World Scientific, Singapore 1989.
- [3] S. Weinberg, *Trans. New York Acad.Sci.* **38**: ,185-201,(1977).
- [4] S. Weinberg, *Phys. Rev. Lett.* **65**1181(1990).
- [5] S. Weinberg, *Phys. Rev. Lett.* **67**3473(1991).
- [6] A. Cabo, S. Peñaranda and R. Martinez, *Mod. Phys. Lett. A* **10**, 2413 (1995).
- [7] M. Rigol and A. Cabo, *Phys. Rev. D* **62**, article no. 074018 (2000) ; hep-th/9909057, (1999).
- [8] M. Rigol “*Acerca de un Estado de Vacío Alternativo para la QCD Perturbativa*”, Graduate Diploma Thesis, Instituto Superior de Ciencias y Tecnología Nucleares, 1999.
- [9] T. Muta, *Foundations of Quantum Chromodynamics*, World Scientific Lectures Notes in Physics - Vol. 5, 1987.
- [10] C. Caso, *et.al.*, *Review of Particle Physics, The European Physical Journal C* **3**, 1 (1998).
- [11] A.A Abrikosov, L.P. Gorkov and J.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, Dover Publications, New York, 1963.
- [12] G. K. Savvidy, *Phys. Lett.* **71B** , 133(1977).
- [13] I. A. Batalin, S. G. Matinyan, y G.K. Savvidy, *Sov. J. Nucl. Phys.* **26**, 214(1977).
- [14] S. G. Matinyan, y G.K. Savvidy, *Sov. J. Nucl. Phys.* **25** (1977) 118; *Nucl. Phys. B* **134** (1978) 539.
- [15] T. Kugo and I. Ojima, *Suppl. Prog. Theor. Phys.* **66** (1979).
- [16] V. Elias , T.G. Steele and M.D. Scadron, *Phys. Rev. D.* **38**(1988)1584.
- [17] V. Elias and T.G. Steele , *Phys. Lett. B* **212**(1988)88.
- [18] A. Cabo and M. Rigol, hep-ph/0008003, (2000), sent to *Nucl. Phys. B*.